GENERALIZED NOMOGRAM FOR FINDING APPROXIMATE TEMPERATURE OF A THERMALLY INSULATED CYLINDRICAL OR FLAT WALL

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A generalized nomogram is obtained from Grover's nomogram [2] by changing the flat wall parameter to a generalized cylindrical wall parameter. Formulas are given for computing this parameter, and examples of calculations with the nomogram are presented.

As shown in [1] the temperature of a thermally insulated cylindrical wall can be calculated from the nomogram for a flat two-layer (metal-insulation) wall [2]. This nomogram becomes universal for the problems mentioned if the flat wall parameter μ is replaced by the generalized parameter

$$\mu^* = k^* + \frac{1}{\mathrm{Bi}^*} + \frac{k^*}{\mathrm{Bi}^*}, \tag{1}$$

where

$$k^* = \Delta k + k f_1, \tag{2}$$

$$\mathrm{Bi}^* = \mathrm{Bi}\, f_2, \tag{3}$$

with

$$\Delta k = \frac{\frac{1}{6}\,\frac{\delta}{r}}{1 + \frac{1}{3}\,\frac{\delta}{r}}, \tag{4}$$

$$f_1 = \frac{1 + \frac{\delta}{r}}{1 + 0.5\,\frac{\delta}{r}}, \tag{5}$$

$$f_2 = \frac{1}{1 + 0.37\,\frac{\delta}{r}}. \tag{6}$$

Here δ/r has the same sign as the increment of the radius in the direction from the heat-transfer surface into the body of the wall.

It should be noted that in calculating a two-layer cylindrical wall Eqs. (2), (4), and (5) do not reflect the change in the total heat capacity of the metal layer due to the difference in curvature of its boundary surfaces. Taking this factor into account necessitates the introduction of the relative thickness of the metal δ_M/δ as an additional parameter. This parameter has a significant effect only for appreciable curvature of the wall and large values of k, the region of small Fourier numbers being the most sensitive. Figure 1 shows the relative error $\varepsilon = (\Theta - \Theta^*)/\Theta^*$ as a function of δ/r for various values of δ_M/δ when $\mathrm{Fo} = 0.1$, $k = 100$, and $\mathrm{Bi} \to \infty$. The quantity Θ is calculated under the assumption

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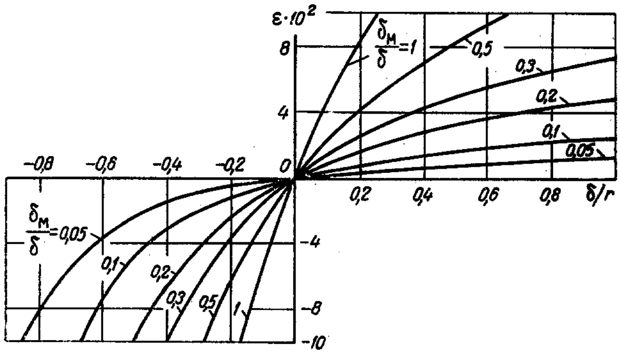

Fig. 1. Relative error $\varepsilon = (\Theta - \Theta^*)/\Theta^*$ as a function of δ/r for various values of δ_M/δ.

$$\delta_M \ll \delta, \ (\delta_M/\delta \to 0). \qquad (7)$$

The temperature Θ^* is calculated by taking account of various values of δ_M/δ.

For values of $\delta_M/\delta \leq 0.1$ the error ε does not exceed 2% in the range $-0.3 \leq \delta/r \leq 0.7$ and is less than 5% for $-0.7 \leq \delta/r \leq 1$. Calculations show that the error decreases as k is decreased. Thus for k = 1 ε is 1.5-2 times smaller, and for k = 0.1 an order of magnitude smaller, than for k = 100. For a uniform cylindrical wall the restriction (7) is of course unnecessary and there is no error connected with this restriction.

Since the nomogram in [2] covers only positive values of μ it cannot be used when $\mu^* < 0$. Equations (1)-(6) show that the smallest value of $\mu^* = -0.25$ corresponds to heat transfer between a uniform solid cylinder and a medium with a boundary condition of the first kind ($\delta/r = -1$, $k = 0$, $Bi \to \infty$).

Our calculations permit the expansion of the nomogram by means of the lines $-0.25 \leq \mu^* < 0$ and the extension and expansion of the range of positive values of the generalized parameter to $\mu^* = 100$. This

TABLE 1. Relative Wall Temperature for Fo = 1

δ/r	—1	0	1	2
k	0	0,1	2	0
Bi	10	5	∞	0,1
μ^*	—0,203	0,32	2,792	21,08
Θ (Exact calculation)	0,0136	0,2841	0,7723	0,9598
Θ Nomogram	0,014	0,28	0,76	0,96

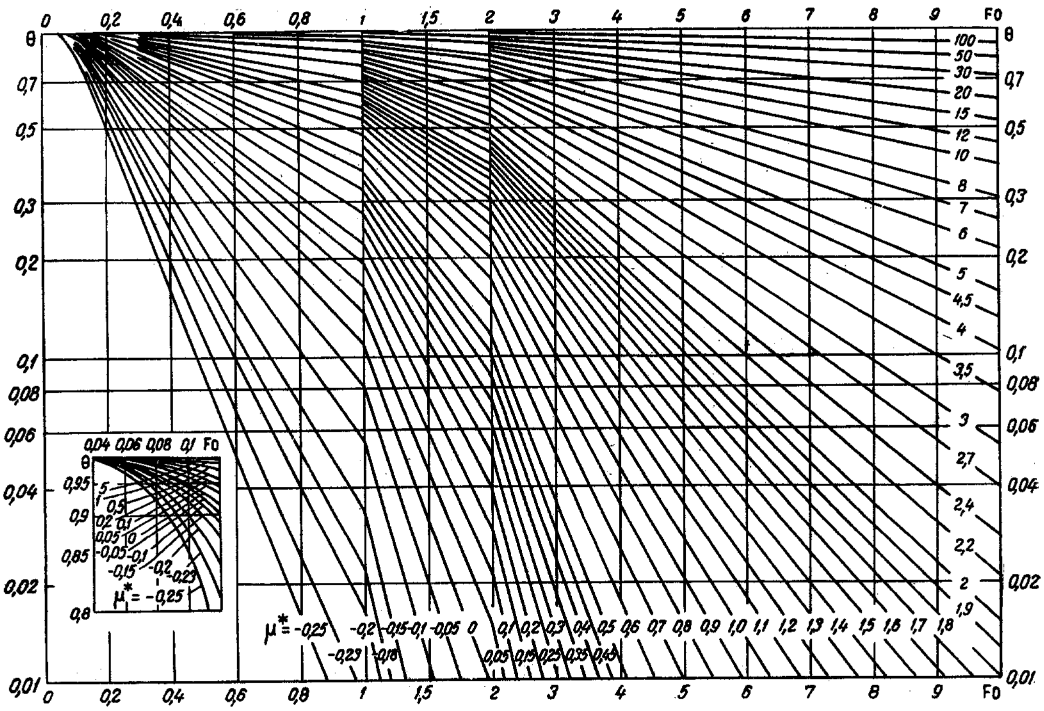

Fig. 2. Generalized nomogram for the approximate calculation of the temperature of a thermally insulated cylindrical or flat wall.

generalized nomogram is shown in Fig. 2. Curves of $\Theta(\mu^*, \mathrm{Fo})$ for small Fo are shown on an enlarged scale in the insert.

The nomogram permits calculations of the relative excess temperature of cylindrical and flat walls up to values of $\mathrm{Fo} \leq 10$. With restriction (7) Eqs. (1)-(6) ensure that the accuracy in determining the temperature in the range $-1 \leq \delta/r \leq 2$ is limited only by the accuracy of interpolation on the nomogram.

Table 1 lists examples of the calculation of the wall temperature from the generalized nomogram for sets of the parameters δ/r, Bi, and k corresponding to various values of μ^*. The values of the temperature obtained by an exact computer calculation are also shown for comparison.

NOTATION

Fo, Bi	are the Fourier and Biot numbers calculated as for a flat wall along the thermal insulation;
r	is the radius of the surface across which heat is exchanged with the medium;
δ	is the thickness of the thermal insulation, or the thickness of the wall if it is uniform;
δ_M	is the thickness of the metal;
k	is the ratio of the total heat capacity of the metal to that of the insulation in a flat wall;
k^*	is the ratio of the total heat capacity of the metal to that of the insulation, corrected for a cylindrical wall;
μ	is the flat wall parameter, $\mu = k + 1/\mathrm{Bi} + k/\mathrm{Bi}$;
μ^*	is the generalized cylindrical wall parameter, $\mu^* = k^* + 1/\mathrm{Bi}^* + k/\mathrm{Bi}^*$;
Bi^*	is the Biot number, corrected for a cylindrical wall;
Δk, f_1, f_2	are, respectively, the correction and the coefficients taking account of the effect of the curvature of the wall on the ratio of the total heat capacity of the metal to that of the insulation and on the heat exchange with the medium;
$\Theta = (T_c - T) / (T_c - T_0)$	is the relative temperature of the wall surface across which there is no heat exchange with the medium;
T_c	is the temperature of the medium;
T_0, T	are the initial and running temperatures of the wall;
Θ^*	is the relative temperature of the wall, calculated by taking account of the variation in curvature within the thickness of the metal layer;
$\varepsilon = (\Theta - \Theta^*)/\Theta^*$.	is the relative error in calculating the temperature.

LITERATURE CITED

1. A. G. Gabril'yants, Inzh.-Fiz. Zh., 14, No. 5 (1968).
2. I. H. Grover and W. H. Holter, "Solution of the transient heat-conduction equation for an insulated, infinite metal slab," Jet Propulsion, 27, 1249 (1957).